Inertial Parameter Estimation of Floating Base Humanoid Systems using Partial Force Sensing

Michael Mistry, Stefan Schaal, and Katsu Yamane

Abstract— Recently, several controllers have been proposed for humanoid robots which rely on full-body dynamic models. The estimation of inertial parameters from data is a critical component for obtaining accurate models for control. However, floating base systems, such as humanoid robots, incur added challenges to this task (e.g. contact forces must be measured, contact states can change, etc.) In this work, we outline a theoretical framework for whole body inertial parameter estimation, including the unactuated floating base. Using a least squares minimization approach, conducted within the null-space of unmeasured degrees of freedom, we are able to use a partial force sensor set for full-body estimation, e.g. using only joint torque sensors, allowing for estimation when contact force measurement is unavailable or unreliable (e.g. due to slipping, rolling contacts, etc.). We also propose how to determine the theoretical minimum force sensor set for full body estimation, and discuss the practical limitations of doing so.

I. INTRODUCTION

We wish for humanoid robots to execute fast, dexterous motion in a robust, compliant, and human-like way. Model-based control methods, which consider the dynamic properties of the robot, are able to specify the necessary control forces required for some desired motion. This makes it possible for proactive control of balance during fast motions, for example, as opposed to reactive correction after some delays. However, these control methods rely heavily on the accuracy of the particular dynamic model used. Obtaining accurate models for model-based control is a significant challenge in robotics, especially for high degree of freedom systems such as humanoid robots.

There has been substantial work done on data driven approaches to inertial parameter estimation of robotic systems [1],[2],[3],[4]. In these works, motion and forces are recorded while the robot executes sufficiently exciting trajectories. Then, inertial parameters (such as mass, center of mass, inertia tensors) can be fit to the observed data. These works have traditionally focused on fixed based systems such as manipulators and industrial robots. However, humanoids and other legged systems, have floating base dynamics which complicate matters for some of the following reasons: 1) the floating base is unactuated, but its inertial parameters must still be identified, 2) torque sensing at all joints, as well as force/torque sensors at all contacts are required, 3) contact

states typically change (e.g. during locomotion), and the identifiability of certain parameters depends on the contact state, and 4) the range of allowable motions can be limited due to balance requirements, constraints, etc.

Because of these complications, it is best to collect as rich a data set as possible from a variety of activities, with several contact conditions (squatting, reaching, locomotion, etc). Ideally our systems would have full force sensing available, which implies force or torque sensing at all prismatic or rotary joints, respectively, and 6 axis force/torque sensing at all contact points. However, due to cost or design limitations, this typically not the case in current humanoid systems. When full force sensing is not available, we would like to have an understanding of what is possible with a reduced set. Recently progress in this area has been made, where it has been shown contact force sensing alone (without any joint torque sensing) is enough for full body parameter estimation [5],[6]. This result is encouraging since the majority of humanoid systems are currently lacking joint torque sensors. However, there are several reasons why we may not want to ignore joint torque sensors in humanoid robots. For example, force/torque sensors may not be available at all contact locations (such as the hands), or may be unreliable (e.g. the foot is slipping or rolling). Additionally, if we are interested in accurate joint tracking, as many model-based controllers are, it may be best to tune parameters to fit local measurements at the specific joints we are trying to control.

In this paper, we attempt to outline some of the theoretical issues involved in inertial parameter estimation of floating base systems, such as humanoid robots. We would like to develop a general framework for least-squares fitting of full-body inertial parameters, using a partial set of force/torque sensors (possibly a combination of joint torque and contact force sensors). We would like to understand how and when a partial subset can be used, which may be useful the design of future systems. Ultimately we would like to be able to obtain accurate models for implementation of model-based controllers such as ones proposed for full-body humanoids ([7],[8],[9],[10],[11],[12],[13]), legged locomotion systems [14], or even passivity control systems that rely on gravity compensation [15].

M. Mistry and K. Yamane are with Disney Research Pittsburgh, 4615 Forbes Avenue, Suite 420, Pittsburgh PA 15213, USA [mmistry, kyamane]@disneyresearch.com

S. Schaal is with the Department of Computer Science and Neuroscience, University of Southern California, Los Angeles, CA, USA and the ATR Computational Neuroscience Laboratories, Kyoto, Japan sschaal@usc.edu

II. FLOATING BASE RIGID BODY DYNAMICS

The floating base framework provides the most general representation of a rigid-body system unattached to the world, and is necessary to describe the complete dynamics of the system with respect to an inertial frame. The system

configuration is represented as:

$$\mathbf{q} = \begin{bmatrix} \mathbf{x}_b^T & \mathbf{q}_r^T \end{bmatrix}^T \qquad (1)$$

where $\mathbf{q}_r \in \mathbb{R}^n$ is the joint configuration of the rigid body robot with n joints and $\mathbf{x}_b \in SE(3)$ is the position and orientation of the coordinate system attached to the robot base, and measured with respect to an inertial frame. Figure 1 illustrates this representation by showing the 6 virtual degrees of freedom attached from inertial frame to the robot base frame.
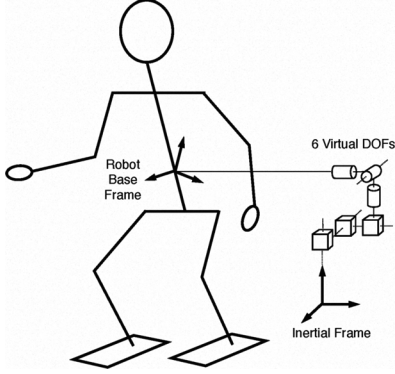

Fig. 1. The base frame attached to the robot is connected to the inertial frame via 6 unactuated virtual DOFs

When the robot is in contact with the environment, the equations of motion with respect to an inertial frame are given by:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{S}^T \tau + \mathbf{J}_C^T(\mathbf{q})\lambda \qquad (2)$$

with variables defined as follows:

- $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n+6 \times n+6}$: the inertia matrix including the floating base
- $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n+6}$: the floating base centripedal, Coriolis, and gravity forces.
- $\mathbf{S} = \begin{bmatrix} \mathbf{0}_{n \times 6} & \mathbf{I}_{n \times n} \end{bmatrix}$: the actuated joint selection matrix
- $\tau \in \mathbb{R}^n$: the vector of actuated joint torques
- $\mathbf{J}_C \in \mathbb{R}^{k \times n+6}$: the Jacobian of k constraints
- $\lambda \in \mathbb{R}^k$: the vector of contact forces

A. Linearity With Respect To Inertial Parameters:

As shown in [16],[17], we can express the complete dynamics of the system to be linear with respect to a set of inertial parameters.

$$\mathbf{K}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\phi = \mathbf{S}^T \tau + \mathbf{J}_C^T(\mathbf{q})\lambda, \qquad (3)$$

where $\phi = \begin{bmatrix} \phi_0^T & \phi_1^T & \dots & \phi_n^T \end{bmatrix}^T$ is the vector of inertial parameters of $n+1$ links (n joints plus the floating base). Each link has 12 parameters, defined as follows:

$$\phi_i = [\, m_i \quad m_i c_{x_i} \quad m_i c_{y_i} \quad m_i c_{z_i} \quad I_{xx_i}$$
$$I_{xz_i} \quad I_{xz_i} \quad I_{yy_i} \quad I_{yz_i} \quad I_{zz_i} \quad f_{c_i} \quad f_{v_i} \,]^T, \qquad (4)$$

where m_i is the mass of link i, $(c_{x_i}, c_{y_i}, c_{z_i})$, is the link's center of mass position, and $(I_{xx_i}, I_{xy_i}, I_{xz_i}, I_{yy_i}, I_{yz_i}, I_{zz_i})$ are the 6 independent components of its inertia tensor. Additionally f_{c_i} and f_{v_i} are coulomb and viscous friction, respectively.

III. INERTIAL PARAMETER ESTIMATION

Since we are able to write the system dynamics as linear with respect to inertial parameters, we can use ordinary least squares to fit parameters to collected data. We move the robot in a sufficiently exciting manner, collecting N sample points along the motion. Each sample point i consists of the joint and base configuration vectors and their derivatives: $(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i)$, and, if full force/torque sensing is available, the vectors of joint torques τ_i and contact forces λ_i. Next we combine sample points by creating stacked matrices in the following manner:

$$\begin{bmatrix} \mathbf{K}(\mathbf{q}_1, \dot{\mathbf{q}}_1, \ddot{\mathbf{q}}_1) \\ \mathbf{K}(\mathbf{q}_2, \dot{\mathbf{q}}_2, \ddot{\mathbf{q}}_2) \\ \vdots \\ \mathbf{K}(\mathbf{q}_N, \dot{\mathbf{q}}_N, \ddot{\mathbf{q}}_N) \end{bmatrix} \phi = \begin{bmatrix} \mathbf{S}^T \tau_1 + \mathbf{J}_{C_1}^T \lambda_1 \\ \mathbf{S}^T \tau_2 + \mathbf{J}_{C_2}^T \lambda_2 \\ \vdots \\ \mathbf{S}^T \tau_N + \mathbf{J}_{C_N}^T \lambda_N \end{bmatrix}, \qquad (5)$$

or in more simple notation as:

$$\overline{\mathbf{K}}\phi = \overline{\mathbf{f}}, \qquad (6)$$

where the bar notation refers to an augmented stack of N matrices, and the following vector of total generalized force:

$$\mathbf{f}_i = \mathbf{S}^T \tau_i + \mathbf{J}_{C_i}^T \lambda_i \qquad (7)$$

is defined for convenience.

Inertial parameters can then be estimated using ordinary weighted least squares:

$$\hat{\phi} = \left(\overline{\mathbf{K}}^T \mathbf{W} \overline{\mathbf{K}} \right)^{-1} \overline{\mathbf{K}}^T \mathbf{W} \overline{\mathbf{f}}. \qquad (8)$$

The parameters computed from (8) are those that minimize:

$$\underset{\hat{\phi}}{\arg\min} \left(\sum_{i=1}^N \left(\mathbf{K}_i \hat{\phi} - \mathbf{f}_i \right)^T \mathbf{W} \left(\mathbf{K}_i \hat{\phi} - \mathbf{f}_i \right) \right). \qquad (9)$$

We can also write this expression in the following way (which we will refer to in later discussion):

$$\underset{\hat{\phi}}{\arg\min} \left(\sum_{i=1}^N \left(\mathbf{W}^{\frac{1}{2}} \mathbf{K}_i \hat{\phi} - \mathbf{W}^{\frac{1}{2}} \mathbf{f}_i \right)^2 \right) \qquad (10)$$

Note that because the generalized forces, $\mathbf{f}$, exist on a non-euclidian space, the weight matrix $\mathbf{W}$ must be chosen to be an appropriate metric such that the units throughout (9) are consistent and minimization with respect to least squares can be well defined. This point is especially relevant for any floating base system, since the generalized forces at the base link are represented by a vector of mixed units (a 6-DOF wrench of combined linear force and torque). The parameters ϕ also live on a non-euclidian space, but if we assume $\mathbf{K}$ is full column rank (see the following section), then (8) will be independent of any metric on ϕ. See [18] for a full discussion of these issues.

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A. Unidentifiable Parameters:

Due to the kinematic structure of the robot, potentially some parameters in ϕ will be unidentifiable, or only identifiable within linear combinations with other parameters. In such a case, the matrix $\mathbf{K}$ will not be full column rank, and therefore not invertible by (8). However, this problem can be resolved using a singular value decomposition [1] or finding the minimal set of identifiable inertial parameters, ϕ_B [19] and rewriting (6) as:

$$\overline{\mathbf{K}}_B \phi_B = \overline{\mathbf{f}}, \tag{11}$$

where $\mathbf{K}_B$ is full column rank.

Note that which parameters are unidentifiable will depend on the contact state of the robot. For example, if a foot is on the ground and not moving (with respect to the inertial frame), it will be impossible to identify its inertia tensor.

B. Physical Consistency:

Although the least squares equation minimizes (9), it may be the case that certain parameters in $\hat{\phi}$ are not physically consistent (having a negative mass for example). In this case, it is possible to project $\hat{\phi}$ into a physically consistent subset (see [4],[20]).

IV. INERTIAL PARAMETER ESTIMATION WITH PARTIAL FORCE SENSING

A. Estimation from Contact Forces Only:

If joint torque sensors are not available, [5] has shown it is possible to estimate the same set of parameters as in (6), but only using contact force measurement. If we expand the matrices in (3):

$$\begin{bmatrix} \mathbf{K}_b \\ \mathbf{K}_r \end{bmatrix} \phi = \begin{bmatrix} \mathbf{0} \\ \tau \end{bmatrix} + \begin{bmatrix} \mathbf{J}_{C,b}^T \\ \mathbf{J}_{C,r}^T \end{bmatrix} \lambda, \tag{12}$$

we can use the just the top half of (12) to fit parameters:

$$\overline{\mathbf{K}}_b \phi = \overline{\mathbf{J}_{C,b}^T \lambda}. \tag{13}$$

In [6] it is proven that the least squares solution to (13) can estimate the same inertial parameters as the full sensor case.

We note that (13) is identical to premultiplying each data point in (6) with $\mathbf{S}_b$:

$$\overline{\mathbf{S}_b \mathbf{K}} \phi = \overline{\mathbf{S}_b \mathbf{f}}, \tag{14}$$

where we define:

$$\mathbf{S}_b = \begin{bmatrix} \mathbf{I}_{6 \times 6} & \mathbf{0}_{6 \times n} \end{bmatrix}, \tag{15}$$

B. Estimation from Joint Torque Sensors Only:

As a dual to (12), we can also write the complete dynamics of the system without contact forces. We take the QR decomposition of $\mathbf{J}_C^T$:

$$\mathbf{J}_C^T = \mathbf{Q} \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix}, \tag{16}$$

where $\mathbf{Q}$ is an $(n+6) \times (n+6)$ orthogonal matrix and $\mathbf{R}$ is an $k \times k$ upper triangular matrix of full rank. If we premultiply (3) by $\mathbf{Q}^T$, we have:

$$\mathbf{Q}^T \mathbf{K} \phi = \mathbf{Q}^T \mathbf{S}^T \tau + \begin{bmatrix} \mathbf{R} \lambda \\ \mathbf{0} \end{bmatrix}. \tag{17}$$

In [13] we show that the upper and lower portions of this equation are decoupled, i.e. we are able to write the full system dynamics using only the bottom portion that does not depend on contact forces:

$$\mathbf{S}_u \mathbf{Q}^T \mathbf{K} \phi = \mathbf{S}_u \mathbf{Q}^T \mathbf{S}^T \tau, \tag{18}$$

where

$$\mathbf{S}_u = \begin{bmatrix} \mathbf{0}_{(n+6-k) \times k} & \mathbf{I}_{(n+6-k) \times (n+6-k)} \end{bmatrix}. \tag{19}$$

We can then fit parameters with the following equation:

$$\overline{\mathbf{S}_u \mathbf{Q}^T \mathbf{K}} \phi = \overline{\mathbf{S}_u \mathbf{Q}^T \mathbf{f}} \tag{20}$$

C. Estimation with a General Reduced Sensor Set:

As it was possible to remove joint torque sensing, or contact force sensing, and still obtain an estimate of inertial parameters, it is possible, to some degree, to generalize the procedure in order to use a specified reduced sensor set, which may be a portion of joint torque sensors combined with a portion of contact force sensors. This can be very useful when the robot has inadequate sensing, for example force/torque sensors at the feet but not the hands (as is typically the case with modern humanoid robots), or only a few joint torque sensors in key locations. The critical issue when using an arbitrary reduced set of sensors, is whether or not the remaining sensors are capable of accurately representing the complete motion of the floating base link.

We segment and reorganize the vectors τ and λ and matrices $\mathbf{S}$ and $\mathbf{J}_C$ into those to be used for parameter estimation: (τ_m, λ_m) and those not to be used: (τ_x, λ_x). We then rewrite (3) as:

$$\mathbf{K} \phi = \begin{bmatrix} \mathbf{S}_m^T & \mathbf{J}_{C,m}^T \end{bmatrix} \begin{bmatrix} \tau_m \\ \lambda_m \end{bmatrix} + \begin{bmatrix} \mathbf{S}_x^T & \mathbf{J}_{C,x}^T \end{bmatrix} \begin{bmatrix} \tau_x \\ \lambda_x \end{bmatrix}$$
$$= \mathbf{A}_m^T \begin{bmatrix} \tau_m \\ \lambda_m \end{bmatrix} + \mathbf{A}_x^T \begin{bmatrix} \tau_x \\ \lambda_x \end{bmatrix} \tag{21}$$

Next, compute the null space basis of $\mathbf{A}_x$ (via a singular value decomposition or row reduction method), which we call $\mathbf{V}_x$. This matrix is defined by the following relationship:

$$\mathbf{A}_x \mathbf{V}_x \mathbf{V}_x^T \xi = \{ \mathbf{0}, \quad \forall \xi \in \mathbb{R}^{n+6}, \tag{22}$$

and the operator $\mathbf{V}_x \mathbf{V}_x^T$ is the null space projection of $\mathbf{A}_x$.

Essentially, we want to be able to conduct parameter estimation within this reduced dimensional null space. In order to be able to do so, we must make sure that key information regarding the dynamics of the system is not lost when projecting into this subspace. As a consequence of the work of [6], we know that the dynamics of the floating base link alone contains critical information. Because the floating base link is the root of the kinematic tree, the total force/torque at this link represents the combined force/torques of all other

links. Additionally, dynamic information regarding this root link is only contained within the forces of this link. Thus, the floating base link is the only link that contains dynamic information of all links, making it possible to estimate inertial parameters for all links, based on the forces/torques at this single link alone (see [6] for a full proof). Therefore, when we project a given generalized force vector into a reduced dimensional subspace, we need to make sure that dynamic information pertaining to the floating base link is retained. We decompose the generalized force vector, $\mathbf{f}$, from (6) into its base and joint components:

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_b \\ \mathbf{f}_r \end{bmatrix} \tag{23}$$

and project this vector into a the null space of $\mathbf{A}_x$:

$$\mathbf{V}_x\mathbf{V}_x^T\mathbf{f} = \begin{bmatrix} \mathbf{V}_{x,b} \\ \mathbf{V}_{x,r} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{x,b}^T & \mathbf{V}_{x,r}^T \end{bmatrix} \begin{bmatrix} \mathbf{f}_b \\ \mathbf{f}_r \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{V}_{x,b}\mathbf{V}_{x,b}^T & \mathbf{V}_{x,b}\mathbf{V}_{x,r}^T \\ \mathbf{V}_{x,r}\mathbf{V}_{x,b}^T & \mathbf{V}_{x,r}\mathbf{V}_{x,r}^T \end{bmatrix} \begin{bmatrix} \mathbf{f}_b \\ \mathbf{f}_r \end{bmatrix}. \tag{24}$$

Thus, in order to retain information contained in $\mathbf{f}_b$ through the projection of $\mathbf{V}_x\mathbf{V}_x^T$, we at least need the sub-matrix $\mathbf{V}_{x,b}\mathbf{V}_{x,b}^T$ to be full rank, or equivalently $\mathrm{Rank}(\mathbf{S}_b\mathbf{V}_x) = 6$. In such a case, our reduced sensor set will be sufficient for estimating ϕ. We can then fit parameters using the following equation:

$$\overline{\mathbf{V}_x^T\mathbf{K}}\phi = \overline{\mathbf{V}_x^T\mathbf{f}} \tag{25}$$

We note that (25) is the general form of (14) and (20). Indeed, $\mathbf{S}_b^T$ is the null space basis of $\mathbf{S}$, and $\mathbf{Q}\mathbf{S}_u^T$ is the null space basis of $\mathbf{J}_C$. Of course, $\mathrm{Rank}(\mathbf{S}_b\mathbf{S}_b^T) = 6$ in all cases, satisfying the requirement for base link identifiability. Additionally, $\mathrm{Rank}(\mathbf{S}_b\mathbf{Q}\mathbf{S}_u^T) = 6$ when all degrees of freedom of the the floating base can move independently within the null space of $\mathbf{J}_C$.

D. Minimal Sensor Set

We can now reason as to what can be the minimal possible force sensor set for full body parameter estimation. This number will depend on the contact state of the robot. The critical issue is to be able to sense all external forces in some way, either through direct contact force sensing or through joint torque sensing. We can distinguish between constrained and unconstrained branches of the kinematic chain of the robot (Figure 2), where the base link is considered to be the root of the tree. Unconstrained branches are those chains with no external forces. In this case, we know that all forces applied by this branch to the base link will be due to inertial motion alone. However, in a constrained branch, we will need to distinguish between the inertial motion of the branch and external forces applied to the branch. This is possible if we have at least the same number of sensors located along this branch as there are linearly independent external forces applied to this branch (the sensors must also be able to sense in linearly independent directions). For example, a typical humanoid robot with two feet on the ground (6 linearly independent constraints for each leg), will require at least

6 linearly independent sensors on each leg. This can be one force/torque sensor per foot, or 6 joint torque sensors along the leg (all aligned to be non-parallel), or a combination of the two.
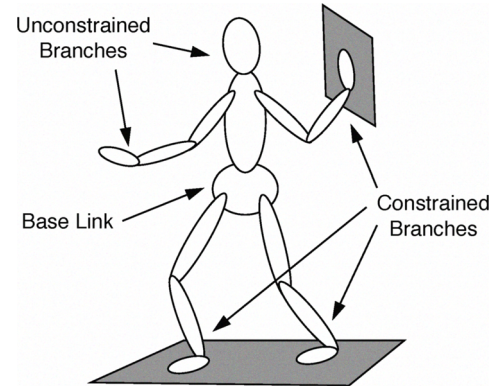

Fig. 2. Constrained and Unconstrained branches of a humanoid robot's kinematic chain. As long as we can sense the forces at the base link, unconstrained branches require no additional force sensing. However, since constrained branches have external forces applied, force sensing is required in order to distinguish between inertial and external forces.

E. Practical Issues for Control

Although it is theoretically possible to estimate full-body parameters using a minimal set of force sensors, for practical reasons we may not want to do so. Different sensor sets will result in different parameter values, as each set attempts to minimize a different function. Fitting parameters according to (25) will minimize (10) with $\mathbf{W}^{\frac{1}{2}} = \mathbf{V}_x^T$. As a consequence of the null space projection, certain forces, which may be important for a particular control task, will not be considered in the minimization procedure. For example, estimations using only contact forces may result in poor joint tracking. If highly accurate center-of-pressure placement is required, it is very useful to include foot force/torque sensing. On the other hand, including too many sensors can possibly degrade performance. For example, it is reasonable to assume that balancing may improve if noisy upper-body sensors are excluded. When deciding what sensors to use for estimation (or what sensors to design into your robot), it is important to consider the control tasks.

V. EVALUATIONS

For a basic evaluation of the parameter estimation theory, we will use the SL simulated bipedal robot, modeled after the lower half of the Sarcos Humanoid robot (Fig. 3). The simulated robot has 2×7 DOF legs and a 1×2 DOF torso, for a total of 16 actuated DOFs with torque sensing. Each foot is represented by 4 point contacts, and floor contact is simulated using a spring-damper model. In the simulator, the integration loop runs at 1000Hz and the feedback control loop at 500Hz.
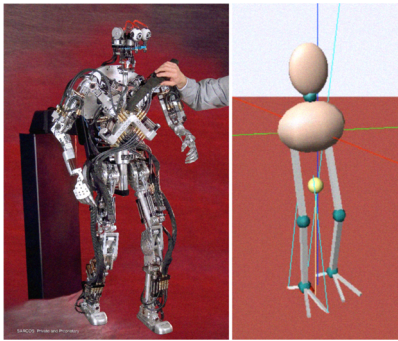
an adequate job of all-around force and torque prediction. Not surprisingly, *Contact Only* performs well only on base-link predictions, and not joint torques. Interestingly, although *Joints Only* excels within the projected subspace where it was trained, it performs poorly when extrapolating outside that subspace. However, we may only be interested in controlling our robot within this subspace, (e.g. when we do not care about contact forces), and thus this technique should yield the best prediction. Finally, we should mention that with the addition of more training data with greater variety of motions, we would expect all these techniques to improve in performance.
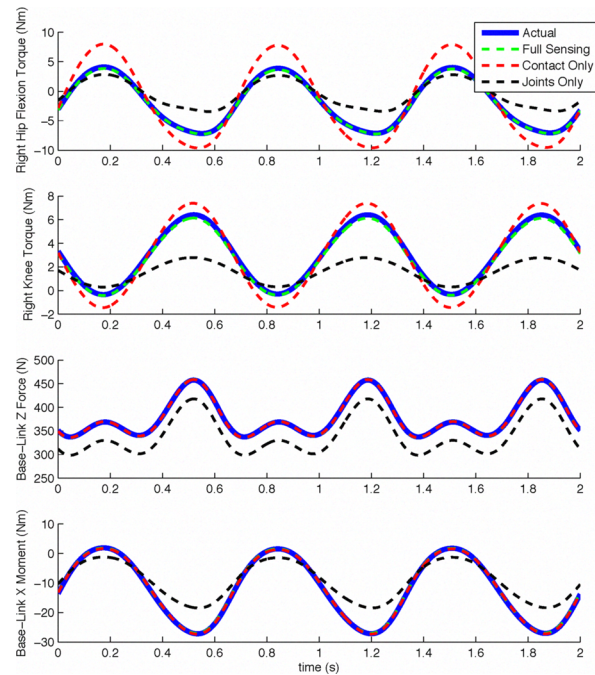
We record a training data set for inertial parameter estimation consisting of two types of motion: 1) a series of squatting motions at various speeds, with two feet in contact at all times, and 2) a series of motions where squatting is superimposed with a side-to-side sway and alternatively lifting each foot off the floor. The complete training set is made of 30 seconds each of squatting at 0.5, 1.0, 1.5, 2.0, 2.5, 3.0 and 3.5 Hz, and combined squatting/stepping motions at 0.0, 0.5, 1.0, 1.5, and 2.0 Hz squatting and 1/6 Hz side-to-side sway with stepping pattern. The 6 minutes of training data is filtered and down-sampled to a 200 Hz sampling rate for a total of 72,000 data points. We evaluate the training on 6 seconds of 1.5 Hz squatting. We will use three different parameter estimation techniques. For *Full Sensing*, we use (6) with full joint torque sensing and full contact force/torque sensing at the feet. For *Contact Only*, we use (14), with only contact force/torque sensors at the feet. Finally for *Joints Only* we use (20) with all 16 torque sensors at each joint.

Figures 4 and 5 show some prediction results obtained by each of the 3 methods. The top two graphs of Fig. 4 show the predicted values of the generalized force vector ($\mathbf{M\ddot{q}} + \mathbf{h}$) for the right hip flexion/extension joint and the right knee joint respectively. The lower two graphs of Fig. 4 show the prediction components of the base-link's z force (the direction of gravity) and the base-link's x moment (forward-backward rotation). Figure 5 shows two representative components of the predicted generalized force vector projected into the null-space of the constraint Jacobian ($\mathbf{S}_u\mathbf{Q}^T(\mathbf{M\ddot{q}} + \mathbf{h})$). It is within this null-space where *Joints Only* performs its regression. Finally Figure 6 shows the RMS prediction error values over the complete 6 second evaluation. The top chart shows the average joint torque prediction errors. The following charts show the base-link force, base-link moment, and projected generalized force predictions.

As expected, these results show that *Full Sensing* does

Fig. 4.   Predictions of the generalized force vector ($\mathbf{M\ddot{q}} + \mathbf{h}$) using the methods of Full Sensing, Contact Force Sensing Only and Joint Torque Sensing Only, and compared with the actual forces in blue. The top 2 graphs are the components of the right hip flexion/extension and right knee joints respectively, and the lower 2 graphs are the base-link's z force (in the direction of gravity) and the base-link's x moment (front-back sway).

VI. CONCLUSION

In this work we have outlined some of the theoretical issues regarding inertial parameter estimation for floating base humanoid systems. We have proposed a framework for full body parameter estimation using a subset of force/torque sensing. We have also discussed the minimal requirements
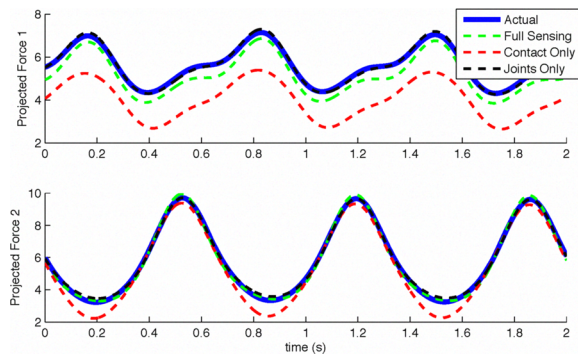
Fig. 5. Predictions of the three methods when projected into the null-space of the constraint Jacobian $(\mathbf{S}_u \mathbf{Q}^T (\mathbf{M}\ddot{\mathbf{q}} + \mathbf{h}))$. Shown are two representative components. The method of Joints Torque Sensing Only excels here since its regression was performed within this subspace.
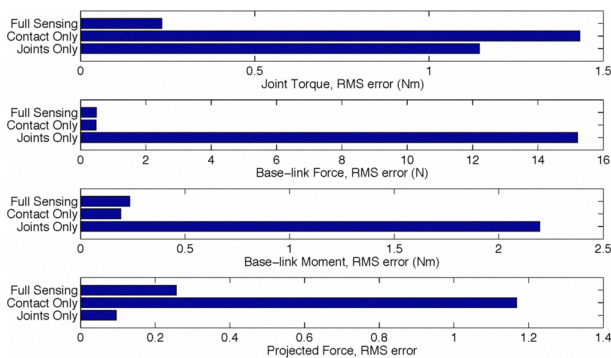

Fig. 6. Top graph is the total joint torque RMS prediction error. Middle two graphs are base-link force and torque prediction respectively. The bottom is the generalized force prediction within the null-space of the constraint Jacobian.

for full parameter estimation. We are actively working on the parameter estimation of our Sarcos humanoid, for the purpose of model-base control (Figure 3). Although full joint torque sensing, and force/torque sensing at the feet are available, the high dimensionally of this system (40 DOF including floating base) as well as practical considerations such as sensor noise and bias, may not warrant the inclusion of all sensors for estimation. For example, if we are only interested in joint motion and not contact force control, then we maybe able to achieve higher joint control accuracy by estimating without contact force sensors. Ultimately it is critical to consider the control task at hand when evaluating which sensors to include in inertial parameter estimation.

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